Statistics

Lecture 8

Feb 19-8:47 AM

SG 14

Data

1) Qualitative

Non-Numerical

2) Quantitative

Numerical

1) Discrete

Countable

2) Continuous

Measureable

Jan 22-4:31 PM

Let x be a discrete random variable with

Prob. dist. $P(x)$.

1) $0 \le P(x) \le 1$

2) $\sum P(x) = 1$

3) $P(x) = 0 \iff$ Impossible event

4) $P(x) = 1 \iff$ Sure event

5) $0 < P(x) \le .05 \iff$ Rare event

what is prob. dist?

Prob. dist. gives the prob. of all possible outcomes.

1) It could be in a form of a table/chart

2) It could be in a form of a graph

3) It could be by certain formula

4) It could be by direct def./formula of probabilities

Jan 22-4:34 PM

ex: Use the chart below

x	$P(x)$
1	.2
2	.5
3	.3

1) Verify $\sum P(x) = 1$ ✓

$\sum P(x) = .2 + .5 + .3 = 1$ ✓

2) Find $P(x \ge 2)$

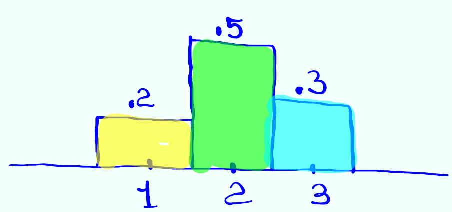
$= .5 + .3 = \boxed{.8}$

3) Find $P(x \le 2) = .2 + .5 = \boxed{.7}$

4) Draw Prob. dist. Histogram

$x \to$ Midpoint

$P(x) \to$ Rel. F.



Jan 22-4:40 PM

Consider the chart below for x with prob. dist. P(x).

x	$P(x)$
1	.15
2	.25
3	.40
4	.20

1) Find $P(x=4)$

$P(x=4) = 1 - [.15 + .25 + .40]$

Total Prob. is 1.

$= 1 - .8$

$= \boxed{.20}$

2) $P(x=1 \text{ or } x=3) = P(x=1) + P(x=3)$

$= .15 + .40 = \boxed{.55}$

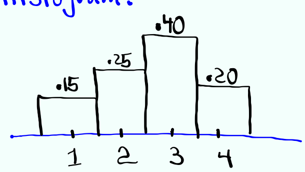
3) $P(x \geq 2) = .25 + .40 + .20 = \boxed{.85}$ ✓

$= 1 - P(x=1) = 1 - .15 = \boxed{.85}$ ✓

4) Draw Prob. dist. Histogram.

$x \to$ Midpoint

$P(x) \to$ Rel. F.



Jan 22-4:45 PM

A piggy bank has 2 dimes & 3 Nickels.

Take 2 Coins, No replacement

Sample Space

NN $\to$ 10¢

ND, DN $\to$ 15¢

DD $\to$ 20¢

$P(10¢) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = .3$

$P(15¢) = 2 \cdot \frac{3}{5} \cdot \frac{2}{4} = \frac{12}{20} = .6$

$P(20¢) = \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20} = .1$

Total	P(Total)
10¢	.3
15¢	.6
20¢	.1

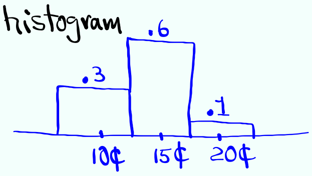
1) $\sum P(T) = 1$

$.3 + .6 + .1 = 1$

2) $P(10¢ \text{ or } 20¢) = .3 + .1$

$= \boxed{.4}$

3) Draw Prob. dist. histogram



Jan 22-4:52 PM

Complete the chart below for discrete random variable x with prob. dist. $P(x)$.

x	$P(x)$	$xP(x)$	$x^2P(x)$
1	.3	.3	.3
2	.5	1.0	2.0
3	.2	.6	1.8

1) Verify $\sum P(x) = 1$

$.3 + .5 + .2 = 1$ ✓

2) Find $\sum xp(x)$

$= .3 + 1.0 + .6 = 1.9$

3) Find $\sum x^2 p(x) = .3 + 2.0 + 1.8 = \boxed{4.1}$

4) Compute $\sum x^2 p(x) - \left(\sum xp(x)\right)^2$

$= 4.1 - (1.9)^2 = \boxed{.49}$

5) Find $\sqrt{\text{Last answer}} = \sqrt{.49} = \boxed{.7}$

2nd (−) Enter

Jan 22-5:00 PM

4 Females, 6 Males, choose 3 different People.

FFF
FFM
FMF
FMM
MFF
MFM
MMF
MMM

Sample Space

Let x be # of Females

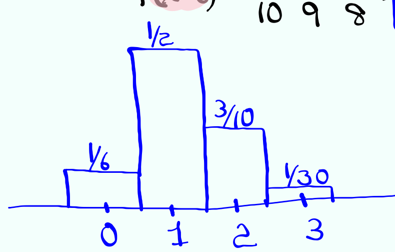
$x=3$ $\quad P(x=3) = \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} = \boxed{\frac{1}{30}}$

$x=2$ $\quad P(x=2) = 3 \cdot \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{6}{8} = \boxed{\frac{3}{10}}$

$x=1$ $\quad P(x=1) = 3 \cdot \frac{4}{10} \cdot \frac{6}{9} \cdot \frac{5}{8} = \boxed{\frac{1}{2}}$

$x=0$ $\quad P(x=0) = \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} = \boxed{\frac{1}{6}}$

# F	P(# F)
3	1/30
2	3/10
1	1/2
0	1/6



Jan 22-5:09 PM

Mean μ (mu)

Variance σ^2 (Sigma^2)

Standard deviation σ (Sigma)

$\mu = \sum x\,p(x)$

$\sigma^2 = \sum x^2 p(x) - \mu^2$

$\sigma = \sqrt{\sigma^2}$

x (L1)	$P(x)$ (L2)	$x\,p(x)$	$x^2 p(x)$
1	.3	.3	.3
2	.5	1.0	2.0
3	.2	.6	1.8

1) $\sum p(x) = 1$

$.3 + .5 + .2 = 1$ ✓

2) $\mu = \sum x\,p(x)$

$= .3 + 1.0 + .6 = \boxed{1.9}$

3) $\sum x^2 p(x) = .3 + 2.0 + 1.8 = \boxed{4.1}$

4) $\sigma^2 = \sum x^2 p(x) - \mu^2$

$= 4.1 - 1.9^2 = \boxed{.49}$

5) $\sigma = \sqrt{\sigma^2}$

$= \sqrt{.49}$

$= \boxed{.7}$

$x \to$ L1

$P(x) \to$ L2

STAT → CALC

1:1-Var Stats

List : L1

FreqList : L2

Calculate

$\mu = \bar{x} = 1.9$

$\sigma = \sigma_x = .7$

$n = 1$

For σ^2

VARS 5: Statistics 4: σ_x

x^2 Enter .49

Jan 22-5:38 PM

Remember the 2 Coin example

Total	P(Total)
10¢	.3
15¢	.6
20¢	.1

Total → x → L1

P(Total) → P(x) → L2

STAT → CALC

1:1-Var Stats

Use L1 & L2

Find σ^2

VARS 5: Statistics

4: σ_x x^2 Enter

9

$\mu = \bar{x} = 14$

$\sigma = \sigma_x = 3$

$n = 1$

Jan 22-5:50 PM

Remembe # Female example

# Females	P(# Females)
3	1/30
2	3/10
1	1/2
0	1/6

Females → x → L1

P(# Females) → $P(x)$ → L2

$\boxed{\text{STAT}}$ $\boxed{\rightarrow}$ CALC

$\boxed{\text{1:1-Var Stats}}$
L1 & L2

$\mu = \bar{x} = 1.2$

$\sigma = \sigma_x = .748$

$n = 1$

Find σ^2 in reduced fraction

$\boxed{\text{VARS}}$ $\boxed{\text{5: Statistics}}$ $\boxed{4: \sigma x}$

$\boxed{x^2}$ $\boxed{\text{MATH}}$ $\boxed{\text{1:}\blacktriangleright\text{Frac}}$ $\boxed{\text{Enter}}$

$\sigma^2 = \frac{14}{25}$ ✓

Jan 22-5:56 PM

Complete the chart below for discrete random variable x with prob. dist. $P(x)$.

x (L1)	$P(x)$ (L2)
1	.2
2	.1
3	.2
4	.3
5	.2

1) Find $P(x=1)$

$= 1 - [.1 + .2 + .3 + .2] = \boxed{.2}$

2) Find $P(2 \le x \le 4)$

$= .1 + .2 + .3 = \boxed{.6}$

3) $\mu = 3.2$

4) $\sigma = 1.4$

5) $\sigma^2 = \frac{49}{25}$ Reduced fraction

$\mu \approx 3$, $\sigma \approx 1$

Usual Range "95% Range"

$\mu \pm 2\sigma$

$= 3 \pm 2(1) \Rightarrow \boxed{1 \text{ to } 5}$

Jan 22-6:02 PM

Expected Value → Mean → μ

I Sold 20 tickets, \$10 each.

1 ticket randomly Selected

owner of ticket gets a Calculator worth \$120.

Net gain	P(Net gain)	
10 − 120	1/20	win
10 − 0	19/20	$\overline{\text{win}}$

For the house

Net gain → X → L1

P(Net gain) → P(X) → L2

$\boxed{\text{STAT}}$ $\boxed{\rightarrow}$ CALC

$\boxed{\text{1: 1-Var Stats}}$

L1 & L2

$E.V. = \mu = \bar{x} = \$4/\text{TKT}$

$\$10(20 \text{ TKTS}) - \$\text{Calc.} = 200 - 120 = 80$

$\frac{\$80}{20 \text{ TKTS}} = \$4/\text{TKT}$

Jan 22-6:10 PM

Ally is going on a trip.

she pays \$50 for luggage insurance policy.

Any damages, Airline pays \$1000.

Prob. of any damage is known to be .2%

Find expected Value per policy sold by the airline.

Net gain	P(Net gain)	
50 − 1000	.2% = .002	damage
50 − 0	99.8% = .998	$\overline{\text{damage}}$

For airline

net gain → X → L1

P(Net gain) → P(X) → L2

1-Var Stats with L1 & L2

$\boxed{E.V. \rightarrow \mu = \bar{x} = 48}$

$\sigma = \sigma_x = 44.677$

$\sigma^2 = 1996$

Jan 22-6:17 PM

Pay me \$5

Draw one card from a standard deck of playing cards.

If you draw an Ace → I give you \$50

" " " a face → " " " \$10

Any other card → I give you nothing

Set up the chart for the house

Net	P(Net)	
5 − 50	4/52	Ace
5 − 10	12/52	face
5 − 0	36/52	any other card

Net → L1

P(Net) → L2

E.V. = $\mu = \bar{x}$ Per bet

= −1.15 House loses \$1.15/play

SG 14 & SG 15 ✓

Jan 22-6:26 PM

SG 16

Binomial Prob. Dist.

1) we have n independent events.

2) Each event has only two outcomes.

P(Success) = p P(Failure) = q

$p + q = 1$, $q = 1 - p$

p & q remain unchanged for all n events.

3) x → # of Successes

$n - x$ → # of failures

$$\boxed{P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}}$$

$x = 0, 1, 2, 3, \ldots, n$

Jan 22-6:43 PM

Consider a binomial Prob. dist. with $n=5$ and $P=.4$. $\to q=1-P=.6$

$n-x \to 5-3=2$

$P(x=3)$

$P(x)={}_nC_x \cdot p^x \cdot q^{n-x}$

$= {}_5C_3 \cdot (.4)^3 \cdot (.6)^2 = \boxed{.2304} \approx \boxed{.230}$

5 $\boxed{\text{Math}}$ PRB $\boxed{{}_nC_r}$ 3 $\boxed{\times}$ .4 $\boxed{\wedge}$ 3 $\boxed{\times}$ .6 $\boxed{\wedge}$ 2 $\boxed{\text{Enter}}$

Jan 22-6:48 PM

Consider a binomial prob. dist. with $n=10$ and $p=.6 \to q=.4$

$P(x=7)$

$P(x)={}_nC_x \cdot p^x \cdot q^{n-x}$

$= {}_{10}C_7 \cdot (.6)^7 \cdot (.4)^3 = \boxed{.215}$

Jan 22-6:55 PM

Crystal is taking a quiz.

There are 20 True/False questions.

She is going to make random guesses.

$n=20$
$p=.5$
$q=.5$

P(she gets exactly 12 correct answers)

$P(x=12)$

$P(x)={}_nC_x \cdot p^x \cdot q^{n-x}$

$={}_{20}C_{12} \cdot (.5)^{12} \cdot (.5)^{8} \approx \boxed{.120}$

Jan 22-6:59 PM

Tyler is taking a multiple-choice exams.

40 questions, Each question has 4 choices but only one correct choice.

Tyler is making random guesses.

$n=40$ $p=\frac{1}{4}=.25$ $q=\frac{3}{4}=.75$

P(He guesses 15 Correct Ans.)

$40-15=25$

$P(x=15)$

$P(x)={}_nC_x \cdot p^x \cdot q^{n-x}$

$={}_{40}C_{15} \cdot (.25)^{15} \cdot (.75)^{25}$

$\approx \boxed{.028}$

Rare event

Jan 22-7:04 PM